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On the limitations of the Brinkman–Forchheimer-extended Darcy equation

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In this work, several philosophical points with respect to the momentum equation in a porous medium are analyzed. We show that several erroneous/irrelevant issues were put forward in previous work. A porous medium/clear fluid interface is best dealt with by the Brinkman–Forchheimer-extended Darcy formulation and the continuity of velocities and stresses at the interface. The effect of porosity variation is not required for a high-porosity medium but should be considered for a dense porous medium.

Keywords: porous media, non-Darcian effects

Introduction

The purpose of this work is to address a topic discussed in an earlier work by Nield (1991). In that work Nield argues about several philosophical points with respect to the momentum equation in a porous medium. In this work, we show that several erroneous/irrelevant issues were put forward by Nield.

Nield (1991) dealt with an important and classical problem involving the fluid mechanics of the interface region between a porous medium and a fluid layer. Earlier Vafai and Kim (1990a) obtained an exact solution to this problem using a Brinkman–Forchheimer-extended Darcy equation (generalized momentum equation). Assuming two-dimensional (2-D), steady-state, isotropic, incompressible, homogeneous flow through a fluid-saturated porous medium, Vafai and Tien (1981) obtained the governing momentum equation based on local volume averaging and matched asymptotic expansion as follows:

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mu_{\text{eff}}\nabla^2\mathbf{v} - \frac{\mu_f}{K}\mathbf{v} - \frac{F\rho}{\sqrt{K}}|\mathbf{v}|\mathbf{v} \tag{1}$$

Nield questioned the validity of including macroscopic flow development and the macroscopic viscous shear stress term (or Brinkman term). He concluded that, for flow in a dense porous medium, the first term in Equation 1 can be omitted. He suggested dropping the Brinkman term and using the Beavers–Joseph boundary condition for flow at a porous/fluid interface. We review his comments on these issues. For the sake of appropriateness, we will follow the same order as that in Nield (1991).

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Brinkman term

It should be mentioned that in the field of "fluid flow and heat transfer in porous media," as in the field of "turbulence," some of the information has been obtained from an intuitive basis rather than a formal approach. Kolmogrov microscales, flow and thermal scaling, buckling and eddy formation, and coherent structures constitute a few of these intuitively based ideas in the field of turbulence. This type of approach is expected to lead to increasingly realistic future models for the prediction of technologically important turbulence or porous media problems. Certaintly for a developing field, such as turbulence or porous media, all things cannot be resolved in one attempt. All sections of useful information are very important in the development of any of these fields. There is also a need for some systematic assumptions and approximations in these fields. Vafai and Kim's (1990a) approach is based on Vafai and Tien's (1981) original approach in which the shape of the averaging volume is chosen for physical interpretation of the relevant averaged quantities. Therefore, our averages, in effect, will correspond to the line averages of the physical quantities in the transverse direction that is normal to the flow. Thus, our results are valid, as mentioned in the manuscript, for a 2-D flow. This and subsequent discussions point out Nield's flawed arguments on this issue. Indeed, by choosing a cylindrical volume, we can satisfy the criteria for choosing three distinctive length scales in porous media; i.e., $d \ll h \ll L$, where d is some microscopic characteristic length representative of the distance over which significant variations in the point velocity take place; h is a characteristic length for the averaging volume; and L is some macroscopic characteristic dimension representative of the process under consideration.

It should be noted that the empirical information employed to obtain Equations 14 and 15 of Vafai and Tien (1981) concerns specific physical terms in the fundamental averaged transport equations and is very different from the global empirical relations as given in Equations 3 and 5 in that reference. Therefore, the shape of the averaging volume gives a direct physical interpretation of the relevant averaged

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quantities. That is why the inclusion of the Brinkman term is valid for a 2-D flow field, as was substantiated in Vafai and Tien(1981). Nield has missed the crucial physical significance of this point in his work, as he argues that the inclusion of the Brinkman term is valid for a three-dimensional (3-D) flow field. Without the 2-D restriction, the physical phenomena described by the Brinkman term would be flawed and without any physical basis. It should be noted that the form of the equations used in this work constitutes the most comprehensive equations for porous media, and its derivation can be made more rigorously by applying the method of matched asymptotic expansion, giving exactly the same final result as given in Vafai and Tien (1981).

Furthermore, even smaller scales than those cited above have been encountered in a number of other fluid mechanics problems; for example, the boundary-layer thickness for a high Reynolds number flow close to the leading edge of an external boundary, or the initial interface region between two high-speed fluids (no porous media), or the entire constant boundary-layer thickness for a high-speed disk rotating in a low-kinematic viscosity fluid (covers a significant number of practical fluids) near the ground. Another example pertains to the shock layer. For a shock layer only about 10 mean free paths thick, as measured by Sherman (1955), there was considerable doubt that the Navier-Stokes or the continuum approach altogether is not valid. However, agreement with the experiment was found to be excellent, and attempts to improve the analysis with kinetic theory have not resulted in any substantial improvement.

In another argument, Nield (1991) mentions that if the variable porosity effects are not accounted for, then use of the Brinkman term leads to no benefit. This statement is simply wrong. As mentioned earlier, various porous media have very high porosities. For media with such high porosities (e.g., in the upper 90 percent), porosity variations are not significant; whereas, the boundary effects are very significant. Importance of boundary and inertia effects for constant porosity as well as variable porosity aspects are analyzed in detail in Vafai and Tien (1981, 1982), Vafai and Thiyagaraja (1987), Beckermann et al. (1986, 1988), Kladias and Prasad (1988), David et al. (1988), Vafai (1984), Amiri and Vafai (1994), as well as several other works. The importance of the Brinkman-extended Darcy flow model has been demonstrated by Tong and Subramanian (1985), Vasseur and Robillard (1987), Lauriat and Prasad (1987), and Prasad et al. (1988), and the importance of Forchheimer-extended Darcy flow model has been demonstrated by Beavers and Sparrow (1969), Bejan and Poulikakos (1984); Poulikakos (1985), Poulikakos and Bejan (1985), and Prasad and Tuntomo (1987).

Nield (1991) also mentions that most natural media have a porosity less than 0.6, giving the impression that, for most practical applications, the porosity is less than 0.6. This is not the case. First, various naturally occurring media have very high porosities, such as lava, sponge-type media and snow. Furthermore, in modeling flow over a number of naturally occurring configurations, a high porosity model must be used. In addition, many such manmade materials as various insulation and Foametals have porosities in the nineties.

It would be instructive to discuss some of Nield's previous works that are relevant to the present investigation. In the paper entitled "The Boundary Correction for the Rayleigh–Darcy Problem: Limitations of the Brinkman Equation" (Nield, 1983), he questions the applicability of the Brinkman equation and mentions "The no-slip condition on rigid boundaries necessitates a correction to the critical value of the Rayleigh–Darcy number for the onset of convection in a horizontal layer of a saturated porous medium uniformly heated from below. It is shown that the use of the Brinkman equation to obtain this correction is not justified because of the limitations of that equation" and "We have shown that although the Brinkman equation is useful in the treatment of flow past a very sparse collection of obstacles, and for flows in porous media where the velocity is constant except in regions near boundaries, it is not generally applicable to flow in porous media."

However, in a later work (Nield, 1984), he realizes that his questioning of the applicability of the Brinkman equation and his conclusion about it are incorrect, and he states "and this led me (Nield, 1983) to question the applicability of the Brinkman equation in the bulk of the porous medium. I now realize that the discrepancy is due, not to the use of the Brinkman equation, but rather to the Galerkin approximation used by Rudraiah et al. (1980). This approximation failed to take into account the boundary layer, which must arise when the Brinkman equation is used." Therefore, the statements made by Nield with regard to the limitations of Brinkman equations were completely retracted by him.

Convective inertial term

The so-called "ideal medium" described by Nield (1991) and used in his discussions has none of the major characteristics of a porous medium. The porous medium considered in the work of Vafai and Kim (1990a, b) and described in more detail in such references as Vafai and Tien (1981) and Vafai (1984) has a random structure and is made of an interconnected structure. The idealized medium considered by Nield (1983) has absolutely none of the cited crucial characteristics; a medium made of identical tubes of uniform cross section is neither random nor does it have any form of interconnected structure associated with it. In fact, for this medium, the flow field inside each tube is essentially completely independent (after a short entry length) of the flowfield inside the other tubes. That is, there is no interaction at all between flow inside one tube and any other tube! The idealized porous medium described by the author can by no means be considered to be a regular porous medium. In lieu of the above discussion, the subsequent arguments made by Nield (1991) are flawed and irrelevant.

On the basis of these observations, Nield (1991) argues about removal of the convective term in the momentum equation for the porous media. The convective term must be present in a generalized equation of motion for the porous medium. This can be proved through a rigorous derivation of the equation of motion along the lines of what has been presented in Vafai and Tien (1981), coupled with a matched asymptotic expansion analysis as that presented in Vafai and Thiyagaraja (1987). In fact, as explained in Vafai and Tien (1981), it is this convective term that is responsible for the boundary-layer development along, for example, an external surface embedded in a porous medium. As was also shown in this same reference, the hydrodynamic developing length for most practical applications is very small and, in general, can be ignored. This fact was later shown to be true numerically by Kaviany (1985). In addition, the equation used by Nield (1991) for initiating his arguments (Equation 10) is incorrect. He presents this equation on the premise that the forces represented by the right-hand side of Equation 2 in his work are in balance, but this happens only when the boundary layer has been developed; that is, after the short hydrodynamic entry length, after which the convective term can be dropped (as explained in Vafai and Tien, 1981). Furthermore, in substantiating his argument, Nield (1991) again draws upon the flawed idealized model alluded to in the previous comment. Without a convective term, there is no mechanism for development of the flow field, which leads to a physically flawed and unrealistic situation.

In another paper (Nield and Joseph, 1985) the importance of inertial effects are studied by Nield through the use of a momentum equation that includes a quadratic drag term (similar to the form used in Vafai and Tien, 1981). However, in another paper (Joseph et al. 1982) it is mentioned that "In this problem the effects of the presence of a swarm of other spheres is accounted for by the nonlinear 'Forchheimer' drag law rather than the linearized (Darcy) drag law. This leads us directly to the following self-consistent problem of the Brinkman type: (the equation given is quite similar to the one used in Vafai and Tien, 1981 and Vafai, 1984)" and in the same reference it is concluded that "we have formulated a nonlinear theory which is consistent with the available experimental data for flow through porous material. Although our work is formulated for flow through a fixed solid matrix, similar considerations apply to sedimentation problem in which particles fall through a viscous fluid."

Interfacial boundary conditions

Just as there are several turbulence models for modeling of high Reynolds number flow, there are several approaches for modeling flow and heat transfer processes in a porous medium. It is well known that the Darcy flow model cannot predict the viscous effects (from the presence of a boundary), the flow development, and the high-velocity effects. This is why there was a definitive need for extending the Darcy model. For details, we can refer to "Principles of Heat Transfer in Porous Media" (Kaviany, 1991). They arrived at a momentum equation through the use of local volume averaging technique. As pointed out earlier, some systematic assumptions and/or approximations are needed for analysis of heat and fluid flow in porous media.

This is again because of the complex nature of physical phenomena in the pores. Perhaps a good and relevant example of this approach is the treatment of an interfacial region between two different fluids. It is conventional to treat such a region by a mathematically dividing surface of zero thickness with two continuum fluids having uniform properties all the way up to the zero thickness interface. Our knowledge of the phase interface is by no means complete, but there is enough experimental data that show density may be a continuous function of position. Perhaps all of the intensive variables should be considered continuous functions of position in going from one phase to the next. However, there is a disadvantage in taking the apparently more realistic view of a continuous interfacial region; namely, making the problem very complex and to some extent undefined. This accounts for the popularity of the zero thickness models for the phase interface. Yet the experimental effects directly attributable to the behavior of the material in the interfacial region cannot be ignored. For this reason, when the error is significant, a correction term such as a surface tension should be introduced. However, in Vafai and Kim's (1990a, b) works, there is no need for the introduction of such a correction term.

The central theme of Nield's work (1991) can be summarized by one statement. 'Over the pore section of the interface the velocity shear is continuous, but this is not the case over the solid section. In the solid the velocity shear is identically zero but in the adjacent clear fluid it has in general some indeterminate non-zero value. The averaged velocity shears therefore do not match" (270). His statement is simply wrong. In the solid region, the viscosity can be considered to be infinite, so the product of the viscosity and the velocity gradient is an indeterminate nonzero. In other words, we should have the continuity of the shear stress at the interface. Mathematically this can be expressed as follows:

$$\mu_f \left.\frac{\partial u}{\partial y}\right)_f = \mu_s \left.\frac{\partial u}{\partial y}\right)_s \tag{2}$$

Nield's argument was based on matching velocity gradient rather than shear stress at the interface. This is also true for the normal stress.

Along the same lines, in the last paragraph of modeling an interface section of the Nield's (1991) work, he argues against the pressure continuous across the interface, and he refers back to the philosophical arguments about what constitutes an interface. It is understood that the interface is an idealization of a region where the pressure goes through gradual changes. However, interface idealization, in effect, amounts to the introduction of a zero thickness surface. This surface, although it is only a mathematical abstraction, is a very useful and practical way of handling the interface problems. In reality, there are going to be gradual changes over a region of finite thickness rather than zero thickness, but again, this gets us back to the same philosophical questions for which we do not have any satisfactory answers. Furthermore, Nield does not offer any type of solution regarding this part.

Once we treat the interface as a surface with zero thickness, we can rigorously show the continuity of both the normal shear and pressure across the interface. Applying Cauchy's lemma at the interface of media I and II we obtain the following:

$$t^{\mathrm{I}}(\mathbf{n}) = -t^{\mathrm{II}}(-\mathbf{n}) \tag{3}$$

Applying Cauchy's lemma again for medium II we obtain the following:

$$t^{\mathrm{II}}(\mathbf{n}) = -t^{\mathrm{II}}(-\mathbf{n}) \tag{4}$$

Therefore,

$$t^{I}(\mathbf{n}) = t^{\mathrm{II}}(\mathbf{n}) \tag{5}$$

The stress vector can be written as follows:

$$t(\mathbf{n}) = T_{ij} n_j \mathbf{e}_i \tag{6}$$

Using Equation 4 in Equation 3 results in three independent interface conditions for the Cartesian coordinate:

$$T^{\mathrm{I}}_{xy} = T^{\mathrm{II}}_{xy} \tag{7}$$

$$T^{\mathrm{I}}_{yy} = T^{\mathrm{II}}_{yy} \tag{8}$$

$$T^{\mathrm{I}}_{zy} = T^{\mathrm{II}}_{zy} \tag{9}$$

or the above can be represented as follows:

$$\mu_{\mathrm{I}}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]_{\mathrm{I}} = \mu_{\mathrm{II}}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]_{\mathrm{II}} \tag{10}$$

$$-P_{\mathrm{I}} + 2\mu_{\mathrm{I}}\left[\frac{\partial v_y}{\partial y}\right]_{\mathrm{I}} = -P_{\mathrm{II}} + 2\mu_{\mathrm{II}}\left[\frac{\partial v_y}{\partial_y}\right]_{\mathrm{II}} \tag{11}$$

$$\mu_{\mathrm{I}}\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]_{\mathrm{I}} = \mu_{\mathrm{II}}\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]_{\mathrm{II}} \tag{12}$$

It can be shown that Equation 11 can be reduced to

$$P_{\mathrm{I}} = P_{\mathrm{II}} \tag{13}$$

because at the interface

$$\mu_{\mathrm{I}}\left[\frac{\partial v_y}{\partial y}\right]_{\mathrm{I}} = \mu_{\mathrm{II}}\left[\frac{\partial v_y}{\partial y}\right]_{\mathrm{II}} \tag{14}$$

for a two-dimensional configuration (Chen and Chen, 1992). Therefore both forms of the normal stress boundary conditions (eq. 10b) cited in Vafai and Kim (1990a, b) are correct. Furthermore, there appears to be a total misunderstanding on the part of Nield (1991) since Vafai and Kim (1990a, b) did not even directly use this condition when they solved their problem numerically. This is because they used one domain approach in terms of Vorticity–Stream Function–Temperature formulation, which satisfies this condition indirectly.

In addition, a real porous–fluid interface is significantly more complicated than what has been stated by Nield (1991). That is why the traditional matching conditions are used in Vafai and Kim (1990a, b). The author's argument, in reality, is only a subset of problems that we must deal with when we consider the micromechanics of a porous–fluid interface. This is a very complex task, which has not even been successfully handled for regular fluid–fluid interfaces. As it was already *clearly and explicitly* mentioned in Vafai and Kim (1990a), we were "not trying to resolve a philosophical and complex question with respect to the physical nature of the interface" (p. 254). However, the author's central theme falls exactly under one of the subsets of this very issue. We cannot consider this issue from a very limited angle without accounting for all of the other equally important issues. Again, it should be mentioned that an exact solution has been presented in Vafai and Kim (1990a) for a classical problem using the classical interface boundary condition.

Beavers and Joseph (1967) interpreted macroscopically experimental results for parallel flows and indicated that the tangential interfacial velocity is not the same as the Darcian velocity, and there is a discontinuity in the tangential velocity. They presented a semiempirical treatment based on the velocity slip, which can be correlated with the following:

$$\left.\frac{du}{dy}\right)_{y=0} = \frac{\alpha}{K^{1/2}}(U_i - U_D) \tag{15}$$

where α is the slip coefficient. Kaviany (1991) presented a review on the slip coefficient α and showed that it depends on the interfacial location, the particle Reynolds number, the gap size, permeability, the porosity, and surface structure of the porous medium, to name a few. Hence, it is not an easy task to determine the slip coefficient.

Furthermore, it should be noted that the Beavers and Joseph (1967) slip condition was rigorously shown to be derivable from the generalized equation of motion (including both the boundary and inertial effects) in porous media, as shown in Vafai and Thiyagaraja (1987, pp. 1401–1403, in particular Table 1, and Figure 10). Earlier Neale and Nader (1974), in a simplified treatment, recognized that the no-slip matching conditions using the Brinkman formulation yield consistent results with the slip flow using Beavers and Joseph's condition for Poiseuille flow in a channel for $\gamma = \sqrt{\mu_{eff}/\mu_f}$.

The reason why Brinkman–Forchheimer-extended Darcy formulation has gained large popularity is partly because it enables investigators to treat a porous/fluid composite region as a single domain. By considering the regular fluid as a porous medium with large permeability, we can solve only one set of conservation equations for the entire domain. The more important reason for its success lies in that numerically predicted results using this formulation together with conventional boundary conditions at the interface, as in Equations 5–7 agree well with experimental results.

Beckermann et al. (1988) studied numerically using this approach and experimentally natural convection in a vertical fluid enclosure partially filled with a fluid saturated porous medium heated from the side. For various test cells, porous-layer configuration and fluid–solid combinations, the model predictions showed excellent agreement with the experimental measurements. Thermal convection caused by heating from below in a porous layer underling a fluid layer was numerically investigated by Chen and Chen (1992). The motion of the fluid in the porous layer is governed by Darcy's equation with the Brinkman and Forchheimer terms; whereas that in the fluid layer is governed by Navier–Stokes equation. Heat transfer rates predicted by the numerical scheme for depth ratio of 0.1 and 0.2 show good agreement with the experimental results of Chen and Chen. To verify the validity of the mathematical formulation based on Brinkman–Forchheimer-extended Darcy equation (generalized equation) of motion, Kladias and Prasad (1991) conducted experiments for a wide range of governing parameters such as Darcy, Forchheimer, Prandtl numbers, and conductivity ratio. Experimental results were reported for natural convection in a horizontal porous cavity heated from below. They achieved an excellent agreement between the experimental results and numerical predictions with variable porosity. For the range of porosity between 0.375 and 0.468 the Brinkman–Forchheimer-extended Darcy solutions were found to compare well with the experimental results only at low Rayleigh number and Darcy number. For higher values of Rayleigh and Darcy numbers, the Brinkman–Forchheimer-extended model (generalized equation) with variable porosity and variable thermal conductivity in the wall regions compares reasonably well with the experimental data. However, there are many porous materials whose porosities are higher than 0.9, as mentioned earlier. For these materials, the effect of variable porosity is not significant at all and can be neglected.

Conclusion

Some empirical information in the form of constitutive relations is necessary for analysis of convective flow in a porous medium such as turbulence. Although this is a step away from the absolute rigorous treatment, the best alternative is to minimize the number of constitutive assumptions and try to use a constitutive equation that is directly tied down to some well-known experimental results. The Brinkman–Forchheimer-extended Darcy formulation, if not perfect, is the most commonly used equation in this regard. This accounts for the boundary-layer development and macroscopic shear stress, as well as microscopic shear stress and microscopic inertial force.

Numerical results based on this formulation have been shown to agree well with experimental prediction. This equation is also very effective for studying the motion of the fluid in the region, which is partly filled with a porous medium and partly filled with a regular fluid. The Beavers–Joseph slip velocity boundary condition is extremely difficult to implement because the slip coefficient depends upon many parameters. A porous medium/clear fluid interface is best dealt with by the Brinkman–Forchheimer-extended Darcy formulation and the continuity of velocities and stresses at the interface. The effect of porosity variation is not required for a high-porosity medium, but it should be considered for a dense porous medium.

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